Prediction of surge height due to tropical storms for the coast of Bangladesh

M. Mizanur Rahman

Abstract— In this paper an estimate of surge height associated with tropical storms is done for the coast of Bangladesh. For this purpose, a vertically integrated model in cylindrical polar coordinate system is developed. Nested numerical schemes were exercised in this study to save computer memory space and to avoid numerical instability as is appropriate for operating forecasting purpose. Vertically integrated shallow water equations are solved using a semi-implicit finite difference technique. Offshore islands of the whole coastal belt along with coastal bending are incorporated through proper stair step representation. The developed model is applied to estimate surge height at different coastal and island locations of Bangladesh associated with the severe cyclonic storm 'AILA' that hit the coast of Bangladesh recently. The result obtained by the model is found to be satisfactory with observed and reported results obtained through various investigators.

Index Terms— AILA ; Bay of Bengal; Cylindrical polar coordinates; Finite difference method; Shallow water model

1 Introduction

Both tropical storms and the associated surges cause tremendous devastation when they approach the coast. The strong winds destroy houses, uproot trees and damage infrastructure, while sea waves of abnormal amplitude lash the coastal belt causing heavy loss of life and property. There is well-documented account of tropical cyclones hitting the Bangladesh coast and the devastation caused by the associated surges. The shape and structure of the Bangladesh coastline, shallowness of water, offshore islands and the huge discharge from the Meghna basin, low lying coastal areas makes it vulnerable for high surge even with a storm of low intensity [1]. Besides these, head Bay region is a large tidal range area with the highest limit along the Meghna estuary. If a storm approaches the coast at the time of high tide, even worse devastation may take place.

Many analyses on prediction of tide, surge and their interaction have been made all over the world. The Atlantic, the Pacific, the North Sea and the Bay of Bengal are the regions that are mostly covered by these works. In the Bay of Bengal, the pioneering works are due to Das [2] and Flierl and Robinson [3]. Subsequently many works have been done but none of them except [1], [4], [5] and [6] considered the existence of islands along the head of Bay region. But it is known that there is a considerable influence of the offshore islands over the surge intensity along the coastal belt ([4], [6]). Moreover, since these islands are thickly populated, it is necessary to estimate the water levels during the storm period at these islands. Hence the inclusion of the islands in a model is essential in order to incorporate the real situation of the head Bay region.

Considering the funnel shape of the Bay of Bengal region, Roy et al.[7] developed a cylindrical polar coordinate model for operating forecasting purpose. But, the study was conducted using uniform distribution of grid lines in radial direction without ensuring fine mash regulation and so, in the study the representation of coastal and inclusion of island boundaries cannot be accurate. Considering the fact into account Rahman et al.[1] developed a model in cylindrical polar coordinates using nested numerical schemes that ensures finer resolution near the coast than that of the deep sea both along radial and tangential directions. The developed model was found to be suitable for incorporating bending of the coastline and the island boundaries more accurately. This present work is actually a development of the study due to Rahman et al. [1].

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2 Theoretical Foundation

2.1 Basic Equation

A system of Cylindrical Polar coordinates is used with the pole, O, located at $\left(24^0\,\mathrm{N}, 91.75^0\,\mathrm{E}\right)$ and at the level of the undisturbed sea (MSL). One boundary line, Ox, on the MSL and along the east coast of India is considered as the initial line of the coordinate system and Oz is directed vertically upwards. The coordinates of any point P in space are then (r, θ, z) where r is the projection (say OL) of OP upon MSL, θ is the angle which the projection OL makes with Ox, and z is the vertical distance of P from the MSL. The other boundary is inclined to the initial line at an angle 90^0 covering the West Coast of Myanmar. The coast of Bangladesh is well included within the model boundaries. The displaced position of the free surface is given by $z=\zeta(r,\theta,t)$ and the position of the sea floor is given by $z=-h(r,\theta)$. Considering the above coordinate system, the vertically integrated linear shallow water equations, as documented by [1], are given by

$$\frac{\partial \zeta}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left[r(\zeta+h)v_r\right]+\frac{1}{r}\frac{\partial}{\partial \theta}\left[(\zeta+h)v_\theta\right]=0 \qquad (1)$$

$$\frac{\partial v_r}{\partial t}-fv_\theta=-g\frac{\partial \zeta}{\partial r}+\frac{T_r-F_r}{\rho(\zeta+h)} \qquad (2)$$

$$\frac{\partial v_\theta}{\partial t} + f v_r = -\frac{g}{r}\frac{\partial \zeta}{\partial \theta} + \frac{T_\theta - F_\theta}{\rho(\zeta + h)} \quad (3)$$

where,

(v_r, v_θ) = radial and tangential components of the vertically integrated Reynolds' averaged velocity,

f = Coriolis parameter,

g = gravitational acceleration,

(T_r, T_θ) = radial and tangential components of the surface stress,

(F_r, F_θ) = radial and tangential components of the bottom stress,

ρ = density of the sea water,

h = ocean depth from the mean sea level.

2.2 Determination of the Forcing Terms

The forcing terms in (2) and (3) are the Coriolis force, surface wind stress and bottom stress. Among the forcing terms, the Coriolis force can easily be generated from $f = 2\omega \sin\phi$ as the angular speed of the earth rotation (ω) and the latitude of the place (ϕ) are known. The parameterization of the bottom stress F_r and F_θ are given by

$$F_r = \rho C_f v_r (v_r^2 + v_\theta^2)^{1/2} \text{ and } F_\theta = \rho C_f v_\theta (v_r^2 + v_\theta^2)^{1/2} \quad (4)$$

where C_f is known as bottom friction coefficient.

The wind stress is, in general, parameterized in terms of the wind field associated with the storm. Following [1] radial and tangential components of the wind stress are given by

$$(T_r, T_\theta) = C_D \rho_a V_a^2 (-\sin\delta, \cos\delta) \quad (5)$$

where C_D, ρ_a and δ are the drag coefficient, air density, and the inflow angle of the circulatory wind of the storm respectively. The inflow angle as a function of r is taken following [8] and [9]. The circulatory wind field is then generated by various empirical formulae. For the Bay of Bengal region most frequently used formula is due to [10], which is

$$V_a = \begin{cases} V_0 (r_a / R)^{3/2} & \text{for all } r_a \le R \\ V_0 (R / r_a)^{1/2} & \text{for all } r_a > R \end{cases} \quad (6)$$

where V_0 is the maximum sustained wind at the radial distance R and r_a is the radial distance at which the wind field is desired.

2.3 Boundary Condition

For the closed boundary (coastal and island boundaries) the normal component of velocity is taken as zero. The radiation type of boundary conditions are preferred for the open boundaries as they can flow out the disturbance created within the analysis area through the open boundary as a progressive wave. According to [1] the western, eastern and southern boundary conditions are, respectively, given by

$$v_\theta + \left(\frac{g}{h}\right)^{\frac{1}{2}} \zeta = 0 \quad (7)$$

$$v_\theta - \left(\frac{g}{h}\right)^{\frac{1}{2}} \zeta = 0 \quad (8)$$

$$v_r - \left(\frac{g}{h}\right)^{\frac{1}{2}} \zeta = 0 \quad (9)$$

3 Numerical Procedure

3.1 Grid Generation

In order to incorporate coastal bending and boundaries of the small and big offshore islands accurately in a numerical scheme, the grid resolution should considerably be high. But this high resolution is unnecessary away from the coast. Moreover if the high grid resolution is exercised throughout the model domain, it may invite numerical instability with more computing cost. The polar coordinate system has an advantage is that the system itself ensures smaller mesh near the pole and larger mesh away from the pole along the tangential direction. Taking the above facts into account, in this study, the pole of the coordinate system is taken near the coastal belt (Fig.1), fine mesh is ensured near the coastal region along the tangential direction. Along this direction, a pencil of uniformly distributed 45 straight grid lines (including the two boundary lines) are considered through the pole. The angle between any two successive straight lines is taken as $\Delta\theta = 2^0$, so that the mesh size in the tangential direction is uniform. Though the mesh size is uniform, the circular arc distance between any two adjacent grid lines decreases as we move towards the pole and increases as we go away from the pole. Since the pole is situated just north of the coastal belt (Fig.1), the arc distance between any successive grid lines is small near the coast and course away from the coast.

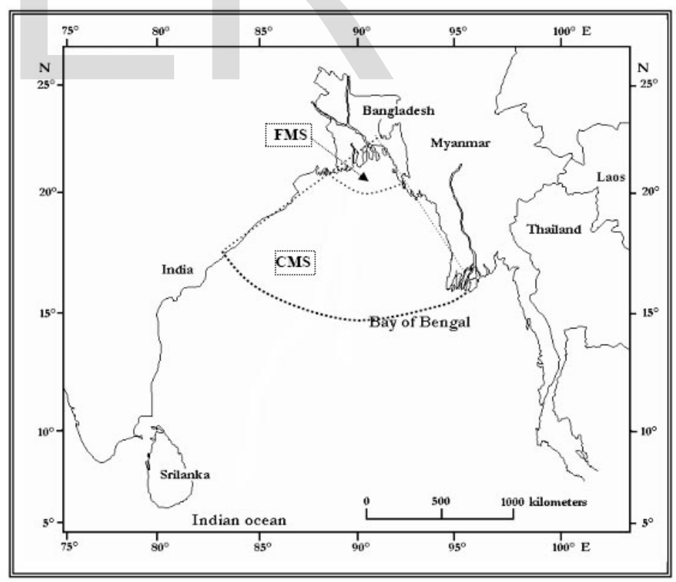

Fig.1 The model area with th boundaries of FMS and CMS

On the other hand, along the radial direction the circular grid lines with centre at O are drawn at a uniform interval of approximately Δr = 18.5 km. There are 45 circle lines in the radial direction. So, other than pole itself, there are 45× 45 grid points in the numerical scheme. Although all the 45 grid lines meet at the pole, for convenience of numerical treatment, it is assumed that there are 45 distinct grid points at the Pole. Thus in the computational scheme there are 46×45 grid points. Of course, no parameter will be computed at the Pole as it is situated at the land, and hence no instability is involved in the computation. The tangential distance (arc distance) between two consecutive grid points varies from zero to approximately 29 km. Since the grid distance in the radial direction is considerably higher, the bends of the coastal belt and the islands could not be incorporated very accurately in the numerical scheme. In this scheme, Bhola island is considered as a part of the mainland while the shape of Hatiya and Sandwip islands are approximate. For more accurate representation, the grid size should be reduced further, and this is done by nesting a fine mesh scheme (FMS) in the above coarse mesh scheme (CMS). The FMS extends from Pole to 18th circular grid line of CMS and lies between same boundary lines of CMS. In the tangential direction two straight grid lines through the Pole have been included between every pair of successive grid lines in CMS. Thus we have a total of 133 grid lines in the tangential direction of the FMS having a uniform grid size of $\Delta\theta = 0.66^0$. In the radial direction, between every pair of consecutive circular grid lines six uniformly distributed circular grid lines have been introduced. So we have a total of 120 circular grid lines in the radial direction of the FMS having a uniform grid size of Δr = 2.6 km approximately. Thus in the FMS there are 120 × 133 grid points and it covers the coastal belt and offshore islands of Bangladesh. The FMS is coupled with the CMS in such a way that CMS is independent, whereas along the open boundary of the FMS the elevation is prescribed from those obtained along the 18th circular grid line of CMS at each time step of the solution process. A typical grid specification and the representation of a fictitious coastal boundary in the numerical scheme are shown in the Fig. 2.

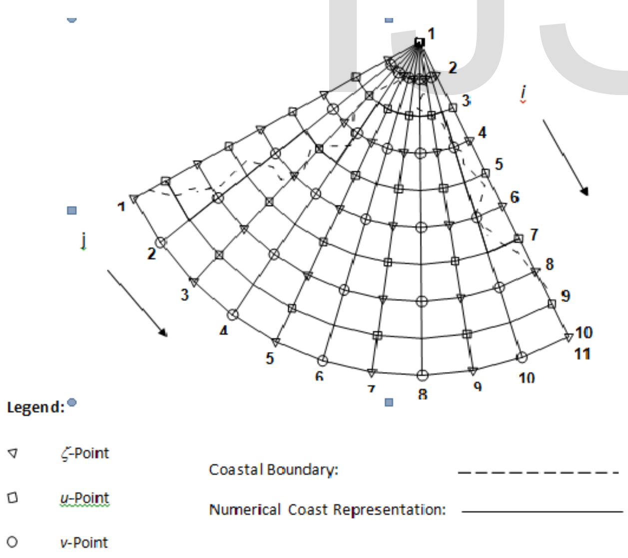

Fig. 2 Numerical grid specification and coastal boundary representation in the numerical scheme.

It may be mentioned that, it is not possible to show clearly the actual grid specification (46×45 and 120 × 133) on the real boundary line, as this needs a very big space. This grid specification is similar to that of [1].

3.2 Discretization of the Governing Equations and Data Initialization for the Model

The model equations given by (1) ,(2), (3) as well as the boundary conditions given by (7), (8), (9) are discretized by finite-difference (forward in time and central in space) and are solved by conditionally stable semi-implicit method using a staggered grid system. For numerical stability, the velocity components in (2) and (3) are modeled in a semi-implicit manner. For example, in the last term of (2) the time discretisation of $v_r\sqrt{\left(v_r^2+v_\theta^2\right)}$ is done as $v_r^{k+1}\sqrt{\left(v_r^2+v_\theta^2\right)^k}$ where k denotes values at present and $k+1$ denotes values at advanced time steps. Along the closed boundary, the normal component of the velocity is considered as zero and this is easily achieved through appropriate stair step representation.

A number of parameters are used in this study. Following [7], the values of the friction coefficient C_f and the drag coefficient C_D are taken as uniform throughout the physical domain, which are 0.0026 and 0.0028 respectively. The necessary meteorological inputs are supplied from Bangladesh Meteorological Department (see, Table 1). The water depth data used in this study is the same what was used in the study of [7]. The initial values of ζ, u and v are taken as zero to represent a cold start. The CFL criterion has been followed in order to ensure the stability of the numerical scheme. The time step 60 seconds is taken to ensures the stability of the numerical schemes. All other values involved in the study has been assumed to have their standard values.

4 Analysis of Results and Model Validation

To analyze and compare model results with observation, the severe cyclonic storm 'AILA' was taken. AILA hit south western coastal districts of Bangladesh on 25 May 2009, killing 190 people, affecting more than 3.9 million people across the 11 coastal districts, disrupting their livelihoods, and destroying infrastructure. The maximum wind speed of AILA was 120 km/h. According to a report from Bangladesh Meteorological Department, a depression formed over the southeast Bay of Bengal at 0900 UTC of 23 May, 2009 and this depression developed into a cyclonic storm AILA at 1200 UTC of 24 May and about 0800 UTC of 25 May the system started to cross West Bengal-Khulna (Bangladesh) coast near Sagar island of India and then moved continuously northwards. At about 1200 UTC of May 25, the central position of the system positioned over Kolkata (India) and adjoining areas of India and Bangladesh. The used tracks of the storm AILA is shown in Fig. 3. Time series about positions of the storm AILA is given in Table 1.

TABLE 1
History of the Storm 'AILA'

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AILA 2009			
Date	Hour	Latitude	Longitude.
2305	0900	$16.00^0 N$	$88.00^0 E$
2305	1500	$16.50^0 N$	$88.00^0 E$
2305	2100	$16.70^0 N$	$88.00^0 E$
2405	0000	$17.20^0 N$	$88.30^0 E$
2405	0600	$17.80^0 N$	$88.60^0 E$
2405	1200	$18.30^0 N$	$88.60^0 E$
2405	1500	$18.60^0 N$	$88.60^0 E$
2405	1800	$18.80^0 N$	$88.60^0 E$
2405	2100	$19.20^0 N$	$88.60^0 E$
2505	0000	$19.40^0 N$	$88.60^0 E$
2505	0300	$20.00^0 N$	$88.60^0 E$
2505	0600	$21.60^0 N$	$88.30^0 E$
2505	1200	$22.90^0 N$	$88.30^0 E$
2505	2000	$24.20^0 N$	$88.50^0 E$
Maximum wind speed : 120 km/h . Maximum radius of sustained wind : 54 km.			

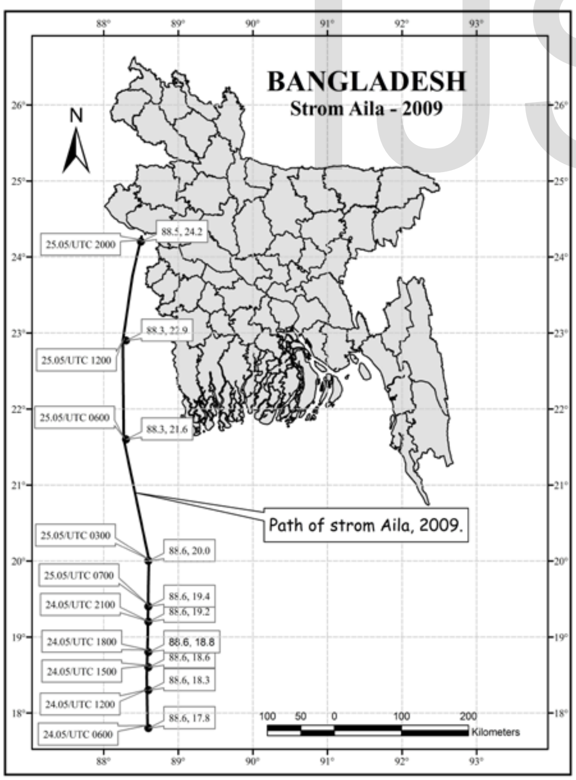

Fig. 3 Observed track (paths) of cyclonic storms AILA.

In this study, the results are presented at some representative coastal and island locations of Bangladesh. Most of the computed results are shown in Figs. 4-7. The results are computed for 80 hours and presented here for the last 48 hours. Fig. 4 depicts the computed surge levels associated with AILA at Hiron Point, Char Jabbar and Sandwip. The peak surge values at those locations can be found to be 2.24m, 2.44m and 2.38m respectively. It may be observed that, the maximum surge level is increasing with time as the storm approaches towards the coast and finally there is recession. At Hiron Point a strong recession is occurred after 07 hrs of 25th May, earlier than in any other locations (Fig. 4). The recession takes place due to backwash of water from the shore towards the sea. The recession reaches up to 1.18m at 2000 hrs of 25th May. The beginning time of recession computed by the model can be found to be in good agreement with the one found in [11]. It may be noticed that the recession at Patharghata, Kuakata, Char Jabbar, Char Chenga, Companigonj, Sandwip, Chittagong and Cox'sbazar began approximately at 0800, 0830, 1200, 1200, 1230, 1230, 1400 and 1500 hrs (Figs. 4, 5, & 6) respectively. Thus the beginning of recession delays as we proceed towards east as is expected. At every location, the peak surge is attaining before the land falls time of the storm. This is expected, as the circulatory wind intensity is highest along the coast when the storm reaches near the coast.

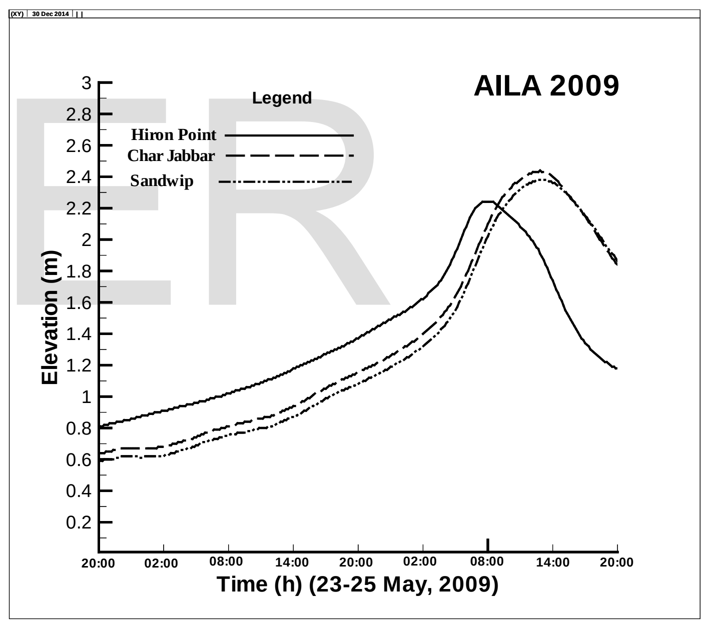

Fig. 4 Computed time series water levels (w. r. t. MSL) at different locations due to surge associated with AILA.

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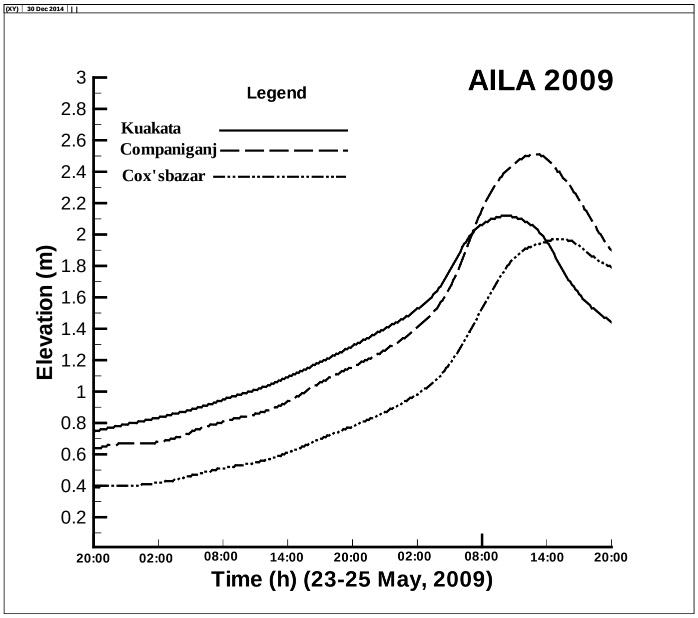

Fig. 5 Computed time series water levels (w. r. t. MSL) at different locations due to surge associated with AILA.

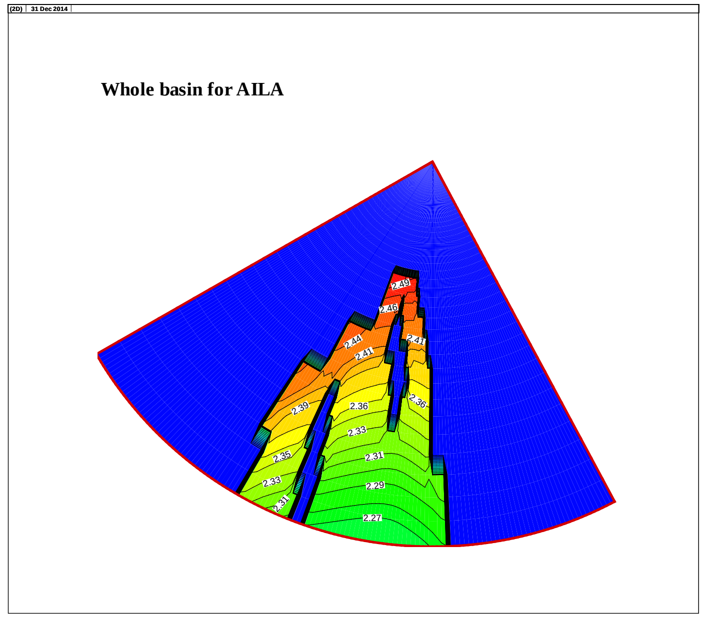

Fig.7 Representation of peak surge levels in the Meghna estuary associated with AILA.

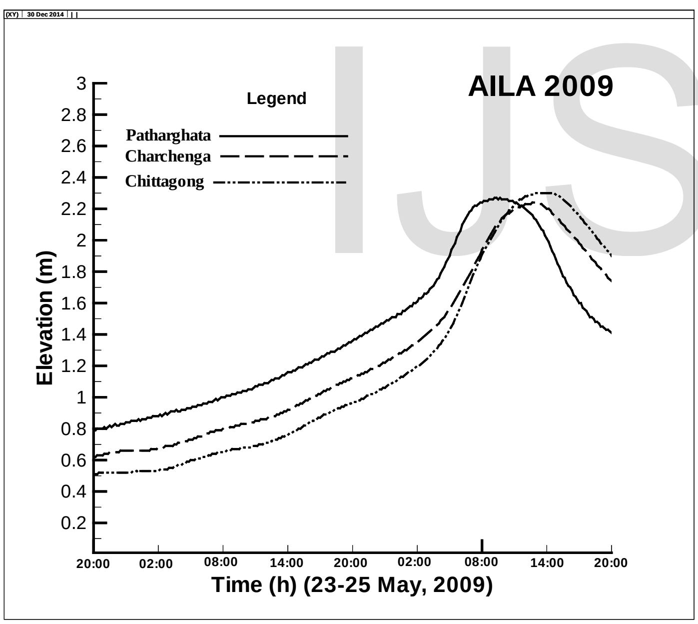

Fig. 6 Computed time series water levels (w. r. t. MSL) at different locations due to surge associated with AILA.

Finally, the contour of peak surge levels associated with AILA along the Meghna estuary region, the region of interest, are shown in Fig. 7.

A comparison of the computed peak surge level at different coastal stations with that of based on the study of [11] and observed peak surge level are given in Table 2. It is evident from the Table 2 that the computed surge heights are almost identical with the observed data.

TABLE 2
COMPARISON OF PEAK SURGE LEVELS AT DIFFERENT LOCATIONS DURING THE STORM 'AILA'

<table>
<tr><th rowspan="2">Coastal locations</th><th colspan="3">Peak surge levels (in meters)</th></tr>
<tr><th>Computed max. surge level</th><th>Max. surge level based on the study of Rahman et al. (2011)</th><th>Observed</th></tr>
<tr><td>Hiron Point</td><td>2.24 m</td><td>1.9 m</td><td rowspan="9">According to Wikipedia website, there was 3 m (10 ft) surge height at the western regions of Bangladesh.

Source: [http://en.wikipedia.org/wiki/Cyclone_Aila]</td></tr>
<tr><td>Patharghata</td><td>2.27 m</td><td>---</td></tr>
<tr><td>Kuakata</td><td>2.12 m</td><td>1.4 m</td></tr>
<tr><td>Char Jabbar</td><td>2.44 m</td><td>2.5 m</td></tr>
<tr><td>Char Chenga</td><td>2.23 m</td><td>1.5 m</td></tr>
<tr><td>Companigonj</td><td>2.51 m</td><td>2.9 m</td></tr>
<tr><td>Sandwip</td><td>2.38 m</td><td>2.3 m</td></tr>
<tr><td>Chittagong</td><td>2.29 m</td><td>2.0 m</td></tr>
<tr><td>Cox'sbazar</td><td>2.01 m</td><td>---</td></tr>
</table>

5 Conclusion

In this study, a surge height forecasting model has developed using nested numerical scheme in cylindrical polar coordinate system. This scheme ensures finer resolution near the coast than in the deep sea both along radial and tangential directions. The model is found to be suitable for incorporating bending of the coastline and the island boundaries more accurately. The model is efficient to compute surge height in the head Bay of Bengal, especially along the coast of Bangladesh. This model may be used for any Bay or Estuary having approximately a funnel shape. This model may be the basis in developing a software to compute water level due to tide and surge along the coast of Bangladesh on real time basis.

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